647510 **MSC2MatC203x** **Seat No :________**

M.Sc.(Maths.) Semester - 2 (*CBCS*) Examination
March/April - 2024 (NEW COURSE)
Topology -2 (Core)

Time: 02:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1.(A) Answer any two questions out of three. **(10)**

(1) Let X be a T_1 space, and let A be a subset of X. Then show that $x \in X$ is a limit point of A if and only if every neighbourhood of x contains infinitely many points of A.

(2) Show that X is Hausdorff space if and only if the diagonal $\Delta = \{x \times x : x \in X\}$ is closed in $X \times X$.

(3) Show that every order topology is Hausdorff.

Q.1.(B) Answer any one question out of two. **(04)**

(1) Define: T_1 Space and T_2 space.

(2) Show that every subspace of Hausdorff space is Hausdorff.

Q.2.(A) Answer any two questions out of three. **(10)**

(1) Show that X is normal if and only if for given a closed set A of X and an open set U containing A, there is an open set V containing A such that $\bar{V} \subset U$.

(2) Show that a regular space with countable basis is normal.

(3) Show that every metrizable space is normal.

Q.2.(B) Answer any one question out of two. **(04)**

(1) Show that a subspace of regular space is regular.

(2) Show that every compact Hausdorff space is regular.

Q.3.(A) Answer any two questions out of three. **(10)**

(1) Let Y be a subspace of X. Then show that Y is compact if and only if each covering of Y by open subset of X contains a finite subcollection covering A.

(2) Sate and prove Lebesgue number lemma.

(3) Show that every compact space is limit point compact.

Q.3.(B) Answer any one question out of two. **(04)**

(1) Show that every compact subspace of Hausdorff space is closed.

(2) State and prove Tube lemma.

Q.4.(A) Answer any two questions out of three. **(10)**

(1) Show that every closed interval of $\mathbb{R}$ is compact.

(2) Let X be a metrizable space. If X is limit point compact, then show that X is sequentially compact.

(3) Let X be a metrizable space. If X is sequentially compact, then show that X is compact.

Q.4.(B) Answer any one question out of two. **(04)**

(1) Let (X, d) be a metric space and $A \subset X$ be a nonempty, then show that $d(x, A) = 0$ if and only if $x \in \bar{A}$.

(2) Give an example to show that every limit point compact space is not compact.

Q.5.(A) Answer any two questions out of three. **(10)**

(1) Show that a metric space (X, d) is complete, if every Cauchy sequence has convergent subsequence.

(2) Show that the Euclidean space $\mathbb{R}^k$ is complete in the square matric ρ.

(3) Let (X, d) be a metric space. Then show that there exists an isometric imbedding of X into a complete metric space.

Q.5.(B) Answer any one question out of two. **(04)**

(1) Show that the closed metric subspace of a complete metric space is complete.

(2) Show that every compact metric space is complete.

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